Spin-Charge Separation and Anomalous Correlation Functions in the Edge States of Quantum Hall Liquids

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First, we have investigated chiral edges of a quantum Hall liquids at filling factor $\nu = 2$. The separation of spin and charge degrees of freedom becomes manifest in the presence of long-range Coulomb interaction. Due to the spin-charge separation the tunneling density of states takes the form $D(\omega) \sim \{-\ln|\omega|\}^{1/2}$. Experimentally, the spin-charge separation can be revealed in the temperature and voltage dependence of the tunneling current into Fermi liquid reservoir. Second, the charge and spin correlation functions of partially spin-polarized edge electrons of a quantum Hall bar are studied using effective Hamiltonian and bosonization techniques. In the presence of the Coulomb interaction between the edges with opposite chirality we find a different crossover behavior in spin and charge correlation functions. The crossover of the spin correlation function in the Coulomb dominated regime is characterized by an anomalous exponent, which originates from the finite value of the effective interaction for the spin degree of freedom in the long wavelength limit. The anomalous exponent may be determined by measuring nuclear spin relaxation rates in a narrow quantum Hall bar or in a quantum wire in strong magnetic fields.

I. INTRODUCTION

Spin charge separation, the transmutation of statistics, and Luttinger liquid behavior are the fundamental properties of interacting 1D electrons [1-3]. Edge electrons of a QH liquid form an interacting 1D chiral liquid [4-9] if the long range Coulomb interaction between edges with opposite chirality is neglected. For short-range interactions the electrons form a Luttinger liquid at fractional filling factors and a Fermi liquid at integer filling factors. The experimental evidence of the power law behavior of Luttinger liquid came from tunneling between two $\nu = 1/3$ edges [10] and between a bulk doped-GaAs normal metal and the abrupt edge of a QH fluid [11].

The separation of spin and charge is an enormous simplification, and it also has important consequences for the low-energy physics. The wave functions and space-time correlation functions factorize into products and the spin and charge collective modes also have different velocities. However, spin charge separation has received little attention in connection with QH edge electrons. The edges can have a composite structure in the fractional case [4,14] and spin charge separation is not expected.

We propose in this paper that the spin charge separation can be revealed in quantum Hall edges states in two ways. First, the tunneling DOS of chiral edge states should reveal spin charge separation at $\nu = 2$, if the long range of the Coulomb interaction is present. The tunneling density of states takes the form

$$D_s(\omega) \sim \frac{1}{\sqrt{\ln\frac{\omega}{|\omega|}}},$$

where the square root in denominator originates from the spin-charge separation. Second, for the non-chiral edge states with the Coulomb interaction between the edges of opposite chiralities we find different crossover behaviors in spin and charge correlation functions, which are another consequence of spin-charge separation. Notably, the spin correlation function in Coulomb dominated regime is characterized by an (non-universal) anomalous exponent, which is due to the novel property of shifting of the guiding center of the Landau level wavefunction with the change in the single-particle quantum number. This paper is organized as follows. In Sec. II, we briefly review the (chiral) Fermi and Luttinger liquid of the edge states and the bosonization methods. In Sec. III, the tunneling density of states of chiral Fermi liquid edge states at $\nu = 2$ is studied. In Sec. IV, the spin and charge correlation functions in the presence of long range Coulomb interaction between edges of different chiralities are calculated. Some new results on the effect of Coulomb interaction in the composite edge states at $\nu = 2/3$ are briefly mentioned in Sec. V, and we conclude with summary in Sec. VI.
II. BOSONIZATION, CHIRAL FERMI AND LUTTINGER LIQUIDS

For the description of the edge states the wavefunction in Landau gauge is most convenient. \((\vec{B} = -(B_y, 0))\)

\[
\psi_n(k, x) = \frac{e^{ikx}}{\sqrt{L}} \chi_n(y - \ell k), \\
\chi_n(y) \propto e^{-y^2/2} H_n(y), \\
E_n = \hbar \omega_c (n + 1/2).
\] (2)

For the Hall bar with width \(W\) (\(\ell\) is the magnetic length),

\[
0 < k < \frac{W}{2\ell}, \quad k = \frac{2\pi p}{L}, \\
0 < p < LWn_B, \quad n_B = \frac{1}{2\pi^2\ell^2}.
\]

In real Hall bar there exists confining potential in \(y\)-direction, and the degeneracy of Landau level is broken. Under smoothly varying confining potential \(\partial_y V(y) \ll \hbar \omega_c/\ell\)

\[
E(n, k) = (n + 1/2)\hbar \omega_c + V(k\ell^2).
\] (3)

Therefore, electrons acquire dispersion in \(k\) and near the edge point where \(\mu = E(n, k)\) massless excitations are supported. The existence of edge states differentiates the quantum Hall states from the ordinary insulators. These low-lying massless excitations are localized near the edges, thus constitute one-dimensional electronic system. For the quantum Hall state at integer filling fraction the low-lying excitations are just non-interacting electrons and they form chiral Fermi liquids \([12]\).

Quantized Hall conductances can be understood very simply in the edge state framework. Suppose there is a chemical potential difference between upper and lower edge. The upper edge carries more current by an amount

\[
I = e\nu\delta n, \quad \delta n = \frac{\delta k}{2\pi}, \quad \nu = \frac{d\omega}{dk}.
\] (4)

The current and conductances are given by

\[
I = \frac{e}{2\pi} \delta \omega_k = GV, \quad G = n \cdot \frac{e^2}{h}.
\] (5)

Therefore, the \textit{integer} Hall conductances can be easily understood while the \textit{fractional} Hall conductances remain a mystery.

For the understanding of the edge state of FQHE the bosonization technique is essential. Let us first consider non-interacting spinless fermions. Linearizing the dispersions near left and right Fermi points, the Hamiltonian becomes \([13]\)

\[
H_0 = v_F \sum_k \left[ (k - k_F) a_{k+q}^\dagger a_k + (-k - k_F) b_{k+q}^\dagger b_k \right]
\] (6)

\[
a_k = \text{Right moving fermion}, \quad b_k = \text{Left moving fermion}.
\]

The density operators are defined by

\[
\rho_R(q) = \sum_k a_{k+q}^\dagger a_k, \\
\rho_L(q) = \sum_k b_{k+q}^\dagger b_k.
\]

What makes the bosonization possible is the following remarkable algebra (U(1) Kac-Moody algebra) which holds only in one dimensional system.

\[
[\rho_R(-q), \rho_R(q')] = [\rho_L(q), \rho_L(-q')], \quad [\rho_R(q), \rho_L(q')] = \delta_{qq'} \frac{2L}{2\pi}, \\
[\rho_R(q), \rho_L(q')] = 0.
\] (7)

The above algebra is also called “axial anomaly” in the field theory context. The essence of bosonization is to express the non-interacting Hamiltonian purely in terms of density operators. We first note the relation \([H_0, \rho_R(q)] = v_F q \rho_R(q)\) and search for the expression for \(H_0\) which can reproduce the above relation. The answer turns out to be

\[
H_0 = \frac{\pi v_F}{L} \sum_q \left[ \rho_R(q)\rho_R(-q) + \rho_L(-q)\rho_L(q) \right].
\] (8)

The fundamental U(1) Kac-Moody algebra can be also realized by canonical bosons. The linkage between canonical boson and density operator is

\[
\rho(x) = -\frac{1}{\pi} \frac{\partial_x \phi}{\partial_t \phi}, \quad j(x) = \frac{1}{\pi} \partial_t \phi.
\] (9)

The second identity is obtained from charge conservation. Because the density operators were originally expressed in terms of fermions the above relation can be designated to be “bosonization formula”. What is remarkable for Eq.(8) is that \(H_0\) is quadratic in density operators. Since major interactions are quadratic in density operators the bosonization methods provide a way to solve many one dimensional system exactly. The final step in bosonization method is the explicit construction of fermion operators in terms of bosons, which is also called “Mandelstam construction”. We will not give the detailed derivation but instead present some heuristic arguments. It is natural to associate the fermion with kinks in bosonic system. Then we need to take care of statistics. Combining two considerations \(\text{Fermion = Soliton} \times \text{Fermi Phase factor}\).

\[
\text{Soliton} = \exp \left[ i\pi \int_{-\infty}^x \Pi(x') dx' \right],
\]

\[
\text{Fermi Phase factor} = \exp \left[ \pm i\phi(x) \right].
\] (10)

\[
\psi_{R,L} = \frac{U_{R,L}}{\sqrt{\pi\alpha}} \exp \left[ \pm ik_F x \mp i\phi(x) + i\theta(x) \right], \\
\theta(x, t) = \pi \int_{-\infty}^x \Pi(x', t) dx'.
\] (11)

\[
\phi(x) = -\frac{i\pi}{L} \sum_{p \neq 0} \frac{e^{-\alpha|p|^2/2 - ipx}}{p} \left[ \rho_R(p) + \rho_L(p) \right] - \frac{N \pi x}{L}, \\
\Pi(x) = \frac{1}{L} \sum_{p \neq 0} \frac{e^{-\alpha|p|^2/2 - ipx}}{p} \left[ \rho_R(p) - \rho_L(p) \right] + \frac{1}{L}, \\
\left[ \phi(x), \Pi(y) \right] = i\delta(x - y).
\] (12)
By doubling species of bosons the above construction can be generalized to spinful fermions.

\[ \psi_{R,L,s} = \frac{1}{\sqrt{2\pi \alpha}} \exp \left[ \pm ik_{F,s} \cdot \mathbf{x} + i\phi_s(x) + i\theta_s(x) \right]. \]

The chiral Fermi liquid edge states can be also understood in boson pictures. The linearization near the right Fermi points gives

\[ H_R = v \sum_k (k - kp) \psi_+^\dagger(k)\psi(k), \]
\[ S = \int dx \, \sigma \cdot \mathbf{\psi} \cdot \mathbf{\psi}^\dagger - i\hbar \mathbf{\nabla} \cdot \mathbf{\psi} \cdot \mathbf{\nabla} \psi. \]

The bosonized Hamiltonian and action are

\[ H = \pi v \int dx \, p^2(x), \]
\[ S = \frac{1}{4\pi} \int dx \, \phi \cdot \mathbf{\nabla} \phi. \]

The Hall conductance can be computed from the bosonized Hamiltonian by linear response procedure.

\[ \langle I(x) \rangle = \int dx' \, D^R(x - x', \omega \to 0) \, V(x'), \]
\[ D^R(x - x', \omega) \approx \int dt \, e^{-i\omega t} \left[ \partial_t \phi(x, 0) \, \partial_{x'} \phi(x', t) \right]. \]

Simple computation shows

\[ \langle I(x) \rangle = \frac{e^2}{h} (V_L - V_R), \]

which is the correct quantized Hall conductance.

For FQHE the fermionic description is inadequate since it always gives integer \( \sigma_{xy} \). However, in bosonic description the fractional Hall conductance can be easily obtained by changing normalization of Hamiltonian appropriately. Specifically,

\[ S = \frac{1}{4\pi} \int dx \, \sigma \cdot \mathbf{\nabla} \phi, \]
\[ H = \frac{e}{4\pi} \int dx \, p^2(x). \]

which gives the required \( G = \nu \frac{e^2}{h} \). Therefore, at least heuristically, we can argue that the above theory is the correct low energy theory of fractional edge states. Another approach from the Landau-Ginzburg theory shows that the above theory is a correct description for Laughlin states \( \nu = \frac{1}{2m+1} \). We also note that the operator \( e^{i\phi} \) has a fractional charge and statistics and it describes Laughlin quasi-particles at edges. Now we are fully ready to study the effect of long range Coulomb interactions on edge states.

### III. Spin-Charge Separation in Chiral Edge States

At filling factor \( \nu = 2 \) there exist both spin up and down edges and they are spatially separated due to the Zeeman splitting, and in general the fermi velocities depend on spin projections. The typical values for Landau level spacing and Zeeman splitting is

\[ h\omega_z = 19.2 \, B(T) \langle K \rangle, \quad E_{\text{Zeeman}} = 0.33 \, B(T) \langle K \rangle. \]

We choose one chiral branch and include only Coulomb interactions between edges of the same chirality. We calculate the DOS of two edges of the same chirality using a bosonization approach. The effective Hamiltonian [5] of edge electrons interacting via the long-range Coulomb interaction has the form

\[ H = \sum_{s, p > 0} \frac{2\pi v_s}{e^2} \rho_{s,p} \rho_{s,-p} + \sum_{s, p > 0} V_s(p) \rho_{s,p} \rho_{s,-p} \]
\[ + \sum_{s, p > 0} V_s(p) \rho_{s,p} \rho_{s,-p}. \]

The intra and inter edge coupling constants are given by

\[ V_s(p) = \frac{2\pi}{e^2} \ln \frac{\nu_s}{|p|}, \quad V_s(p) = \frac{2\pi}{e^2} \ln \frac{\nu_s}{|p|}. \]

The Fermi velocity of the edge with spin \( s \) is \( v_s \). The actual values of the physical parameters appearing in the Hamiltonian must be determined from a microscopic consideration [19]. We form a new basis by taking a canonical transformation

\[ \rho_p = \frac{1}{\sqrt{2}} \left[ \rho_{+p} + \rho_{-p} \right] \quad \sigma_p = \frac{1}{\sqrt{2}} \left[ \rho_{+p} - \rho_{-p} \right]. \]

The Hamiltonian reads

\[ H = \frac{2\pi v_s}{e^2} \sum_{p > 0} \rho_{c,p} : \rho_{p} \rho_{-p} : + \frac{2\pi v_s}{e^2} \sum_{p > 0} \rho_{s,p} : \sigma_{p} \sigma_{-p} : \]
\[ + \frac{2\pi v_s}{e^2} \sum_{p > 0} g \sigma_p \rho_{-p}. \]

The renormalized charge and spin velocities are \( v_{c,p} = v + \frac{2\pi v_s}{e^2} \ln \frac{\nu_s}{|p|} \) and \( v_{s,p} = v + \frac{2\pi v_s}{e^2} \ln \frac{\nu_s}{|p|} \) where \( c^2 = ab \) and \( v = (v_{+1} + v_{-1})/2 \). The coupling between spin and charge modes is given by \( g = v_{c,1} - v_{s,1} \). This Hamiltonian can be diagonalized by the canonical transformation \( \rho_p = \cos \theta_p \tilde{\rho}_p - \sin \theta_p \tilde{\sigma}_p \) and \( \sigma_p = \sin \theta_p \tilde{\rho}_p + \cos \theta_p \tilde{\sigma}_p \), where tan \( 2\theta_p = 2g/\pi(v_{c,p} - v_{s,p}) \). The diagonalized Hamiltonian is

\[ H = \frac{2\pi v_s}{e^2} \sum_{p > 0} \rho_{c,p} : \tilde{\rho}_{p} \tilde{\rho}_{-p} : + \frac{2\pi v_s}{e^2} \sum_{p > 0} \rho_{s,p} : \tilde{\sigma}_p \tilde{\sigma}_{-p} :, \]

where \( \rho_{c,p} \) and \( \rho_{s,p} \) are the renormalized velocities. The bosonization formula for spin \( s \) is given by

\[ \psi_s(x) = \frac{e^{ik_{F,s}x}}{\sqrt{2\pi \alpha}} \exp \left[ -\frac{2\pi}{e} \sum_{p > 0} e^{-a/p} \frac{L \sum_{p \neq 0} e^{-a/p}}{p} \tilde{\rho}_{s,p} \right]. \]

We find that the electron Green's function of spin \( s \) is given by

\[ G_s(x, t) \sim \frac{e^{ik_{F,s}x}}{2\pi \alpha} \]
\[ \cdot \exp \left[ -\frac{1}{2} \int_0^\infty \frac{dp}{p} e^{-a/p} \left( -2 - e^{ip(x-u_{c,p}t)} \right) - e^{ip(x-u_{s,p}t)} \right], \]
\[ -\frac{s}{2} \int_0^\infty \frac{dp}{p} e^{-a/p} \sin(2\theta_p) \left( e^{ip(x-u_{c,s}t)} - e^{ip(x-u_{s,s}t)} \right). \]
For $\omega << \omega_0$ we find

$$D_s(\omega) \sim \frac{1}{\sqrt{\ln \frac{1}{\omega}}} \exp \left[ \frac{sgn}{\pi v_0} \ln \left| \frac{1}{\omega} \right| \right],$$

(25)

where $\omega_0$ is of order $e^2/eW$, $v_0 = 2e^2/(\epsilon \pi)$, and $\omega$ is measured from the Fermi energy. This result should be contrasted to the result $D(\omega) \sim [-1/\ln \omega]$ at $\nu = 1$. The square root in the denominator reflects spin-charge separation [22]. The above result at $\nu = 2$ cannot be obtained in the Hartree-Fock approximation in contrast with $\nu = 1$ spin-polarized case [22].

We will now investigate the DOS when the filling factor is $\nu = 4/3$. The filling factor of spin-up and -down electrons are $\nu = 1$ and $\nu = 1/3$. Including the coupling between the edges within Wen's effective Hamiltonian we find

$$D_4(\omega) \sim \frac{1}{\ln |1/\omega|}, \quad D_{4s}(\omega) \sim |\omega|^{1/2} \frac{1}{\ln 1/\omega^{1/\nu}}.$$  

(26)

These results for DOS are identical to those of isolated edges in the absence of edge coupling, and spin-charge is thus absent at $\nu = 4/3$. It should be stressed that the logarithmic corrections to the DOS are due to the long range of the Coulomb interaction.

Experimentally relevant quantities are the voltage and temperature dependences of the tunneling current. We calculate these quantities for the structure used in a recent experiment [11], where a AlGaAs tunnel barrier is inserted between a 2D electron gas and a 3D doped GaAs. For $eV$ and $T$ less than $\omega_0$ it is possible to obtain simple analytical results, and the results are given below. For $\nu = 2$ and $T = 0$ we find, for both spin channels, that the voltage dependence is given by $I \propto \frac{eV}{\ln z_V}$. The temperature dependence is given by the following approximate interpolating expression between the low and high temperature limits

$$I \propto eV \left( \frac{1}{\ln \max(T,eV/2)} \right)^{1/2}.$$ 

(27)

The dominant tunneling current arises from tunneling into spin up states which are closest to the 3D reservoir. The edge separation between spin up and down electrons is typically of the order of the magnetic length. We have also calculated the voltage and temperature dependences at $\nu = 1/2$.

$$I_4 \sim \frac{eV}{\ln z_V}, \quad I_{4s} \sim \frac{eV}{\ln \frac{1}{\omega}}.$$ 

For comparison, the voltage dependence is calculated for $\nu = 1/3$ down spin edge, $I_4 \sim \frac{eV}{\ln z_V}$.

The temperature dependence is given by the following approximate interpolating expression between the low and high temperature limits

$$I_4 \propto T^{1/\nu} \left[ \frac{eV}{k_B T} \frac{1}{\ln \frac{1}{\omega}} + \left( \frac{eV}{k_B T} \right)^{1/\nu} \frac{1}{\ln \frac{1}{\omega}^{1/\nu}} \right].$$ 

(28)

These results for $\nu = 1/3$ are not identical to the previous results for short range interactions: the Coulomb interaction gives rise to the logarithmic corrections. To observe the logarithmic corrections the experimental data should be fitted to this expression over a wider range of $T$ and $eV$.

A QH chiral liquid at $\nu = 2$ is a rather unique 1D liquid. Although it is not a Luttinger liquid spin charge separation is present provided the electron-electron interactions are given by the long range Coulomb interaction. Our work shows that this effect can be observed in the tunneling current between a bulk doped-GaAs and the abrupt edge of a QH fluid [11].

IV. SPIN-CHARGE SEPARATION IN NON-CHIRAL EDGE STATES

Recent investigations have shown that the long-range Coulomb interaction between edges with opposite chirality brings new effects into these systems [15,16,21,22]. In spin-polarized systems it introduces a cross-over from a Luttinger/Fermi liquid (power law) regime to a regime where the inter-edge Coulomb interaction dominates. A good example is the tunneling conductance at filling factor $\nu \leq 1$ between the quantum Hall edges of opposite chirality. It is described by [21]

$$G \sim \begin{cases} \left( \frac{T}{T_0} \right)^{2(1-1/\nu)}, T > T_0 \\ \left( \frac{T}{T_0} \right)^2 \exp \left\{ -\frac{2}{3\nu} \ln \frac{(T/T_0)^{3/2}}{1} \right\}, T < T_0, \end{cases}$$

(29)

where $T_0$ is the cross-over temperature scale. Similar cross-over exists in the CDW correlations [15,24,25]

$$C(x) \sim \begin{cases} \left( \frac{x}{\ell} \right)^{-2/\nu}, \quad x < W \cos(2k_F x) \exp \left( -\frac{2}{\alpha} \sqrt{\alpha \ln(x^2/\beta)} \right), \quad x > W, \end{cases}$$

(30)

where $\ell$, $W$, and $k_F$ are the magnetic length, the width of the Hall bar, and the Fermi wavevector. In partially spin-polarized QH edges correlation functions are expected to exhibit interesting properties. Since the guiding center of the single particle wavefunction depends on the value of the wavevector, the wavefunctions of spin-up and -down electrons at the Fermi wavevectors are spatially separated. Moreover, spin and charge correlation functions are expected to behave differently since spin and charge separate [22]. We have investigated how spin and charge correlation functions behave as the transverse width of the Hall bar changes. We find that, although charge correlation functions are similar to the result given in Eq.(30), the spin correlation functions acquire anomalous exponents $\alpha_{2k_F}$ and $\alpha_0$. The imaginary part of transverse
spin-spin correlation functions $S(\Omega)$ consist of inter- and intra-branch terms [23]:

$$S_{\text{inter}}(\Omega) \propto \begin{cases} 
\Omega, & \Omega > \Omega_{\text{cr}} \\
\Omega^{\alpha_{2k_F}} e^{-\beta_{2k_F} [\ln \frac{\Omega}{\Omega_{\text{cr}}}]}^{1/2}, & \Omega < \Omega_{\text{cr},} 
\end{cases}$$

(31)

$$S_{\text{intra}}(\Omega) \propto \begin{cases} 
\Omega, & \Omega > \Omega_{\text{cr}} \\
\Omega^{1+\alpha_0} e^{-\beta_0 [\ln \frac{\Omega}{\Omega_{\text{cr}}}]}^{1/2}, & \Omega < \Omega_{\text{cr}}. 
\end{cases}$$

(32)

Here $\nu_{\alpha}(p), d, \beta_{0(2k_F)}$, and $\Omega_{\text{cr}}$ are some appropriate velocity, length, dimensionless constant, and crossover frequency. In the low energy limit the inter-branch contribution dominates over the intra-branch term since $\alpha_{2k_F} < 1 + \alpha_0$. However, the prefactor of the inter-branch term turns out to decay very rapidly as a function of the width of the Hall bar. Consequently, the intra-branch term is more relevant in wide Hall bars. The anomalous exponent $\alpha_0$ goes to zero in the limit where the width of the Hall bar goes to infinity, and an expression similar to Eqs. (1) and (2) is recovered. For a narrow Hall bar or quantum wire in strong magnetic fields the anomalous exponent $\alpha_{2k_F}$ is significant since the edge separation is not negligible compared to the transverse width. The presence of the anomalous exponent in the transverse spin correlation function may be verified experimentally by measuring nuclear spin relaxation rates in a quantum wire in strong magnetic fields. In both expressions for the spin correlation function the power laws in $\Omega$ originate from the spin sector. The anomalous exponent emerges because the difference between spin-up and down electron wavefunctions at the respective Fermi wavevectors makes the effective interaction between the spin degrees of freedom of the opposite edges finite in the long wavelength limit. This effect is absent in zero magnetic field and is unique to partially spin-polarized edges. We adopt the following model for a narrow Hall bar (or equivalently a quantum wire in strong magnetic fields [26]). When the transverse motion is confined by a parabolic potential the single-electron energy levels are given by $E_{n,k} = (n + 1/2)\Omega_0 + \hbar^2 k^2 / 2m^*$, where $m^* = m^* (\Omega_0 / \omega_0)^2$. The subband energy spacing is $\Omega_0 = \sqrt{\omega_c^2 + \omega_0^2}$, where $\omega_c = eB / m^* c$ is the cyclotron energy and $\omega_0$ is the frequency of the harmonic potential. In this model, the degree of spin-splitting, i.e., the distance between the guiding centers of spin-up and -down electrons at the Fermi wavevectors [17], can be easily tuned by the magnetic field since the effective longitudinal mass of the electron depends on the value of the magnetic field [27]. The electronic wavefunction at the Fermi wavevectors $k_{F,r,s}$ of the lowest magnetic subband is given by

$$\phi_{r,s}(x,y) = \frac{e^{i k_{F,r,s} x}}{\pi^{1/4} \tilde{\ell}^{1/2}} \exp \left( \frac{- (y - R^2 k_{F,r,s} x)^2}{2 \tilde{\ell}^2} \right),$$

(33)

where $\tilde{\ell} = (h / m_0 \omega_0)^{1/2}$, $R^2 = \hbar \omega_c / (m_0^2 \omega_0^2)$, $r = R(L)$ for the right (left) branch, and $y$ is the transverse coordinate [28]. Because of the large longitudinal mass at fields where $\omega_c > \omega_0$ all the electrons can be accommodated in the lowest magnetic subband. Each branch $r$ consists of spin-up and -down edges ($s = \uparrow, \downarrow$).

Now we just write down the effective action referring the original paper for details [23].

$$S = \frac{1}{2\pi} \int \frac{d\omega dk}{(2\pi)^2} \left[ (\omega^2 + \nu_{\alpha}(p - \omega))(\nu_{\alpha}(p - \omega) - g_+^2) \phi_{\alpha}(p) \phi_{\alpha} \\
+ (\omega^2 + \nu_{\alpha}(p - \omega))(\nu_{\alpha}(p - \omega) - g_+^2) \phi_{\alpha}(p) \phi_{\alpha} \\
+ 2(-\omega^2 g_+ g_+ - \omega (g_+^2 + g_+^2)) \phi_{\alpha}(p) \phi_{\alpha}, \right.$$  

(34)

where

$$\nu_{p\pm} = \nu_F + \frac{\nu_{\text{intra}}}{\pi} \pm \frac{\nu_{\text{inter}}}{\pi} \frac{x^2}{\pi},$$

$$g_\pm = \delta v_F \pm \frac{\delta v_{\text{inter}}}{\pi}. \quad (35)$$

Finally, we need the explicit expression of electron operators in terms of phase fields [25]

$$\psi_{r,s}(x,y) = \phi_{r,s}(x,y) e^{-i \sqrt{2} \left( \phi_{\alpha}(s + \phi_{\alpha}) + \theta_{\alpha}(s + \phi_{\alpha}) \right)}. \quad (36)$$

It is slightly different from the bosonization formula of a truly 1D system because the left and right edges are spatially separated by the width of the quantum wire. Let us consider the correlation function of the transverse spin operator. The transverse spin operator is

$$\hat{S}^+(x) = \int \frac{dy}{\pi^3} \sum_{r,s=R,L} \psi_{r,1}(x,y) \psi_{s,1}(x,y)$$

$$= C_0(x) e^{-i\sqrt{2} \theta_{\alpha}(x)} \cos \sqrt{2} \phi_{\alpha}(x) + C_{2k_F}(x) e^{-i\sqrt{2} \theta_{\alpha}(x)} \cos \sqrt{2} \phi_{\alpha}(x),$$

(37)

where $C_{0(2k_F)}(x) = e^{-i(\pi_\alpha + \pi_{\text{intra}} + 2k_F \pi_\alpha)/4} e^{-\pi_\alpha / 4k_F^2}$. The first term of Eq.(37) is the intra-branch contribution, and the second term is the inter-branch contribution ($2k_F$ component). Note that the intra-branch spin operator is composed entirely of spin bosons ($\phi_{\alpha}(x)$ and $\theta_{\alpha}(x)$) while the inter-branch spin operator is composed of both the charge $\phi_{\alpha}(x)$ and spin $\theta_{\alpha}(x)$ degrees of freedom. When the off-diagonal elements of the action are negligible the correlation function of the intra-branch spin operator will only reflect the spin degree of freedom.

The inter- and intra-branch terms of the imaginary part of the transverse spin correlation function are given in Eqs. (31) and (4) [31]. The anomalous exponent $\alpha_{2k_F}$ is given by [32]

$$\alpha_{2k_F} = \frac{\nu_F + \frac{1}{2} \ln \frac{\nu_{\text{intra}} W_{\text{intra}}}{\nu_0 W_0}}{\nu_F + \frac{1}{2} \ln \frac{\nu_{\text{intra}} W_{\text{intra}}}{\nu_0 W_0}} - 1.$$

(38)
The other anomalous exponent $\alpha_0$ is

$$\alpha_0 = \left[ \frac{v_F}{v_0} + \frac{1}{2} \ln \frac{\omega}{\omega_0} \frac{W_{\parallel}}{W_{\perp}} \right]^{1/2} + [W_{\parallel} \leftrightarrow W_{\perp}]^{1/2} - 2. \ \ \ (39)$$

The crossover frequency $\Omega_{CR}$ is roughly $v_F/\sqrt{W_{\parallel}W_{\perp}}$ and the anomalous exponents are always non-negative since $W_{\parallel} > 1$. The amplitude of the $2k_F$ component of correlation function $|C_{2k_F}(x)|^2$ is explicitly given by $e^{-2i2k_F^2x^2/\Omega_{CR}^2}$.

V. SPIN-CHARGE SEPARATION IN COMPOSITE EDGES AT $\nu = 2/3$

We briefly mention the spin-charge separation effects in composite edge states at $\nu = 2/3$. The ground state at $\nu = 2/3$ is found experimentally [34] and numerically to be spin-unpolarized for the low magnetic field, and spin-polarized for high field. In the presence of spin-flip tunneling, Zeeman splitting, and random magnetic impurities, the behavior of tunneling conductance through a point contact is found to be very different from the spin-singlet and spin-polarized states [35]. Including the effect of long range Coulomb interaction, the spin and charge conductances take very different form from each other. At very low temperature, the charge conductance decreases exponentially fast, while the spin conductance follows a power law. The exponent of spin conductance consists of the universal part which is determined by ground state structure “K” matrix and the non-universal part determined by the Zeeman splitting and confining potential [36]. In the usual circumstance the universal part always dominates the non-universal part.

VI. SUMMARY

In conclusion, we have shown that the tunneling density of states of chiral edges at $\nu = 2$ acquires a novel logarithmic Coulomb correction owing to the spin-charge separation effect, and the charge and spin correlation functions of partially spin-polarized edge states behave qualitatively different in the presence of long range Coulomb interactions between edges of the opposite chirality. Above effect is unique to 1D systems in strong magnetic fields and is based on the novel property of shifting of the guiding center of the Landau level wavefunction with the change in the single-particle quantum number. In the long wavelength limit the effective interaction for the spin degree of freedom takes a finite value while the effective interaction for the charge degree of freedom is infinitely strong. As a consequence, an anomalous exponent appears in the spin sector. The novel logarithmic Coulomb correction and the presence of the anomalous exponent may be tested experimentally in a narrow and/or QH bar or in a quantum wire in strong magnetic fields.
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REFERENCES

[13] Remember the left and right moving fermions are spatially separated for the edge states in Hall bar.
[27] Since the longitudinal mass enhancement factor is about 20 for $B \sim 7$ T the amount of spin-splitting is significantly large. For a recent experiment see A. R. Goëni, A. Pinczuk, J. S. Weiner. B. S. Dennis, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 70, 1151 (1993).
[30] The Fermi energy of a typical quantum wire in a strong magnetic field is of order 1 meV, which gives $v_F = 7 \times 10^6 \text{cm/s}$. This value should be compared with $v_0 = 1.3 \times 10^7 \text{cm/s}$.
[31] Compared with the power law of the inter-branch part the intra-part has the additional power of 1 in $\Omega$. It stems from the difference of $e^{-2(\phi_\phi \phi)}$ and $e^{-2(\phi_\phi \phi)}$ in the Coulomb regime. The former yields $1/|r|^{1+\hat{\delta}}$, while the latter gives $e^{-\sqrt{\ln |r|}}$.
[32] When the mixed term $\delta H$ is significant the square root must be multiplied by $1 + \delta g + O(\delta g^2)$, where $\delta g = \frac{1}{2} \frac{\partial^2}{\partial \phi^2} \ln \frac{\phi}{\phi_0} \frac{\phi_0}{\phi}$. In quantum wires $\delta g$ is always less than unity, which justifies the expansion in $\delta g$.
[33] When the mixed term $\delta H$ is significant the square root must be multiplied by $1 - \delta g$.